

**Polarization of the final nucleon  
in quasi-elastic neutrino scattering and  
the axial form factor of the nucleon**

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**Abstract**

Measurements of the polarization of the final proton in elastic  $e-p$  scattering drastically changed our knowledge about the electromagnetic form factors of the proton. Here we present our results of the calculation of the polarization of the final nucleon in charged current quasi-elastic neutrino nucleon scattering. Relations which connect the axial form factor with the polarization, the cross section and the electromagnetic form factors of the nucleon are derived. Measurements of the polarization of the nucleon in the high-statistics short baseline neutrino experiments (or in near detectors of T2K and other long baseline experiments) could provide important information on the axial form factor of the nucleon.

## 1 Introduction

Weak and electromagnetic nucleon form factors are important source of information about the structure of the nucleon. Their study is one of the central issues in high energy physics.

The electromagnetic form factors are determined via investigation of elastic scattering of electrons or muons on proton, deuterium and other nuclei. There have been two stages in these studies (see, for example, reviews [1, 2]).

Starting from the famous Hofstadter experiments in the 50's and up to the middle of the 90's, information about the electromagnetic form factors of

the proton and neutron was obtained from measurements of the differential cross section of unpolarized electrons on unpolarized nucleons. The electric  $G_E(Q^2)$  and magnetic  $G_M(Q^2)$  form factors of the nucleon were extracted from these data by the Rosenbluth procedure. From data of the numerous experiments it was found:

1. The proton form factors satisfy the scaling relation:

$$R(Q^2) = \frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)} \simeq 1, \quad (1)$$

where  $\mu_p$  is the total magnetic moment of the proton (in nuclear Bohr magnetons),  $Q^2$  is the squared four-momentum transfer.

2. At relatively small  $Q^2$  ( $Q^2 \lesssim 6 \text{ GeV}^2$ ) the  $Q^2$ -dependence of the proton form factors and the magnetic form factor of the neutron are described by the dipole formula

$$G_M^p(Q^2) \simeq \mu_p G_D(Q^2), \quad G_M^n(Q^2) \simeq \mu_n G_D(Q^2) \quad (2)$$

Here  $\mu_n$  is the magnetic moment of the neutron and

$$G_D(Q^2) = \frac{1}{(1 + \frac{Q^2}{M_D^2})^2},$$

where  $M_D^2 = 0.71 \text{ GeV}^2$ .

In the late 90's series of experiments on measurement of the polarization of the recoil protons in elastic scattering of longitudinally polarized electrons on unpolarized protons started.

In ref. [3] it was shown that measurement of polarization effects in elastic  $e-p$  scattering provides a sensitive way for determination of the electric form factor of the proton. For the ratio of the transverse  $P_\perp$  and longitudinal  $P_\parallel$  polarizations of the proton it was found [3, 4]:

$$\frac{P_\perp}{P_\parallel} = -\frac{G_E^p}{G_M^p} \sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}}, \quad (3)$$

where  $\tau = Q^2/4M^2$  ( $M$  is the nucleon mass) and  $\varepsilon = [1 + 2(1 + \tau)\tan^2\theta/2]^{-1}$  ( $\theta$  is the scattering angle).

Thus, measurement of the ratio  $\frac{P_{\perp}}{P_{\parallel}}$  allows to determine the ratio of the electric and magnetic form factors in a direct model independent way.

Such measurements were done in experiments performed in the Jefferson Lab.: in experiments [5] in the  $Q^2$  range from 0.5 to 5.6  $GeV^2$ , and in the experiment [6] the  $Q^2$  range was extended up to  $Q^2 \simeq 8.5 GeV^2$ . It was established that the ratio  $R$  of the electric and magnetic form factors of the proton was not a constant but decreased linearly with  $Q^2$  starting from  $R \simeq 1$  at  $Q^2 \simeq 1 GeV^2$  and falling down to  $R = 0.28 \pm 0.09$  at  $Q^2 = 5.6 GeV^2$ .

These observations significantly changed the theoretical models for the structure of the nucleon.

Direct information about the axial form factor of the nucleon, which characterizes one-nucleon matrix element of the charged weak current, can be obtained from measurement of the cross sections of the charged current quasi-elastic (CCQE) neutrino processes

$$\nu_{\mu} + n \rightarrow \mu^{-} + p \quad (4)$$

and

$$\bar{\nu}_{\mu} + p \rightarrow \mu^{+} + n, \quad (5)$$

which are the dominant neutrino processes at relatively small neutrino energies ( $E \lesssim 1 GeV$ ).

Starting with the earlier bubble chamber experiments many experiments on the measurements of the cross sections of these processes in a wide range of  $Q^2$  have been done. The results of these experiments are not compatible with each other: from analysis of recent small  $Q^2$ -data significantly larger values of the parameter  $M_A$ , which characterizes the  $Q^2$ -dependence of the axial form factor, have been obtained. Among different reasons for such a disagreement nuclear effects are actively discussed.

The axial form factor of the nucleon is of fundamental importance for the theory. A knowledge of the cross sections of the CCQE processes (4) and (5) in a wide range of energies is extremely important for a correct interpretation of the high-precision neutrino oscillation experiments. Several new experiments (MINER $\nu$ A[7], T2K[8], ArgoNeuT [9]) on a detailed study of CCQE neutrino scattering are going on. In the next Section we will briefly summarize the present day status of the axial form factor of the nucleon.

Measurement of the polarization of the recoil nucleons in the CCQE processes could be a source of an important information on the axial form factor

of the nucleon. Such measurement, like in the electromagnetic case, could change our ideas about the  $Q^2$ -dependence of the axial form factor, about nuclear effects etc. It is worthwhile and timely to consider the possibility for measurement of the recoil polarization in modern short baseline neutrino experiments in which thousands of neutrino events are detected.

In this paper we will present the results of the calculations of the recoil polarization of the nucleon in the CCQE neutrino processes (4) and (5).

## 2 The axial form factor of the nucleon

The determination of the axial form factor of the nucleon is a very challenging experimental problem due to the fact that in neutrino experiments nuclear targets (C, Fe, etc.) are used, the neutrino beams are not monochromatic etc.

In analogy with the electromagnetic form factors the axial form factor is usually parameterized by the dipole formula:

$$G_A(Q^2) = \frac{g_A}{(1 + \frac{Q^2}{M_A^2})^2}. \quad (6)$$

Here  $g_A = 1.267$  is the axial constant, determined from the neutron  $\beta$ -decay data and  $M_A$  is a parameter ( the "axial mass").

The values of the parameter  $M_A$  determined from the data of different experiments, under the assumption that neutrinos interact with a quasi-free nucleon in a nuclei and other nucleons are spectators ( impulse approximation), are quite different.

From analysis of the data on measurements of the cross section of the process  $\nu_\mu + n \rightarrow \mu^- + p$  on deuterium target and of the process  $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$  on proton target it was found [10]:

$$M_A = 1.016 \pm 0.026 \text{ GeV}. \quad (7)$$

This value is in agreement with the value of the parameter  $M_A$  obtained from the data of the NOMAD experiment (carbon target) [11]:

$$M_A = 1.05 \pm 0.02 \pm 0.06 \text{ GeV}. \quad (8)$$

However, from fit of the data of more recent experiments larger average values of the parameter  $M_A$  (with larger errors) were obtained. From the

data of the MINOS experiment (iron target) it was found [12]

$$M_A = 1.26^{+0.12+0.08}_{-0.10-0.12} \text{ GeV}. \quad (9)$$

In the K2K experiment ( $H_2O$  target) it was obtained [13]

$$M_A = 1.20 \pm 0.12 \text{ GeV}. \quad (10)$$

From the high-statistics MiniBooNE experiment (carbon target,  $1.4 \cdot 10^5$  events) it was inferred [14]:

$$M_A = 1.35 \pm 0.17 \text{ GeV}. \quad (11)$$

There could be many different reasons for the disagreement of the average values of  $M_A$  obtained from the data of the different experiments. It could be a problem of systematics and normalization (see [15]). Target nuclei in the different experiments are different. The difference of the values of  $M_A$  could be due to such nuclei effects as interaction of neutrinos with correlated pairs of nucleons (see [16, 17]). Experiments on the study of CCQE neutrino processes were done in different ranges of  $Q^2$ . The difference between different values of  $M_A$  could be a signature that the dipole parametrization (2) is not correct parametrization of the axial form factor in the whole region of  $Q^2$  studied (like in the case of the electromagnetic form factors).

A measurement of the polarization of the recoil protons produced in the CCQE neutrino process  $\nu_\mu + n \rightarrow \mu^- + p$  could change the situation with axial form factor  $G_A(Q^2)$ . Taking into account that in short baseline neutrino experiments thousands of neutrino events are observed it is worthwhile to consider a possibility of measuring of the polarization of the protons by the observation of left-right asymmetry in the scattering of the recoil protons in a neutrino detector.

In the next section we will present our results of the calculation of the polarization of final nucleon in the CCQE neutrino processes.

### 3 Polarization of the final nucleons in CCQE processes

Here we shall present results of the calculations of the polarization of final nucleons in the CCQE neutrino processes (4) and (5).

Process (4) is a charged current process and its matrix element is characterized by the four weak form factors of the nucleon:

$$\begin{aligned} \langle f | (S - 1) | i \rangle &= -i \frac{G_F \cos \theta_c}{\sqrt{2}} N_k N_{k'} \bar{u}(k') \gamma^\alpha (1 - \gamma_5) u(k) \cdot {}_p \langle p' | J_\alpha^{(1+i2)} | p \rangle_n \\ &\times (2\pi)^4 \delta(k + p - k' - p'). \end{aligned} \quad (12)$$

Here  $G_F$  is the Fermi constant,  $\theta_c$  is the Cabbibo mixing angle,

$${}_p \langle p' | J_\alpha^{(1+i2)} | p \rangle_n = N_p N_{p'} \bar{u}(p') (V_\alpha - A_\alpha) u(p), \quad (13)$$

where

$$V_\alpha = \gamma_\alpha F_1^{CC}(Q^2) + \frac{i}{2M} \sigma_{\alpha\beta} q^\beta F_2^{CC}(Q^2), \quad A_\alpha = \gamma_\alpha \gamma_5 G_A(Q^2) + q_\alpha \gamma_5 G_P(Q^2), \quad (14)$$

$F_{1,2}^{CC}$ ,  $G_A$  and  $G_P$  are the CC weak vector, axial and pseudoscalar form factors, respectively,  $k$  and  $p$  ( $k'$  and  $p'$ ) are the initial neutrino and neutron (final muon and proton) momenta,  $N_p = \frac{1}{(2\pi)^{3/2}} \sqrt{2p_0}$  is the standard normalization factor,  $q = p' - p = k - k'$  is momentum transfer,  $Q^2 = -q^2$ .

Under isotopic  $SU(2)$  transformations the weak charged current  $J_\alpha^{1+i2}$  is transformed as the "plus component" of the conserved isovector current. Taking into account that the third component of this isovector is the isovector part of the electromagnetic current (CVC), from isotopic  $SU(2)$  invariance for the weak vector form factors we obtain:

$$F_{1,2}^{CC}(Q^2) = F_{1,2}^p(Q^2) - F_{1,2}^n(Q^2), \quad (15)$$

where  $F_{1,2}^p(Q^2)$  and  $F_{1,2}^n(Q^2)$  are the Dirac and Pauli electromagnetic form factors of the proton and the neutron. These form factors are known at present in a wide region of  $Q^2$  ( see, for example, the review [1, 2]).

From PCAC it follows that the contribution of the pseudoscalar form factor  $G_P(Q^2)$  to the matrix element (14) can be neglected. Thus, from study of the CCQE process (4) an information about the axial form factor  $G_A(Q^2)$  can be obtained.

The matrix elements of the processes (4) and (5) are characterized by the same form factors. In fact from the charge symmetry we have:

$${}_p \langle p' | J_\alpha^{(1+i2)} | p \rangle_n = {}_n \langle p' | J_\alpha^{(1-i2)} | p \rangle_p. \quad (16)$$

The polarization 4-vector of the final proton in process (4) is given by the expression:

$$\xi^\rho = \frac{\text{Tr} \gamma^\rho \gamma_5 \rho_f}{\rho_f}, \quad (17)$$

where  $\rho_f$  is the final spin density matrix. Using the relation

$$\Lambda(p') \gamma^\rho \gamma_5 \Lambda(p') = 2M \left( g^{\rho\sigma} - \frac{p'^\rho p'^\sigma}{M^2} \right) \Lambda(p') \gamma_\sigma \gamma_5 \quad (18)$$

and performing integration over the momenta of the final lepton and nucleon, for the polarization 4-vector of the final nucleon we have:

$$\xi_\alpha = \left( g_{\alpha\beta} - \frac{p'_\alpha p'_\beta}{M^2} \right) \frac{\text{Tr} [\mathcal{N} \Lambda(p) \bar{\mathcal{N}} \Lambda(p') \gamma^\beta \gamma_5]}{\text{Tr} [\mathcal{N} \Lambda(p) \bar{\mathcal{N}} \Lambda(p')]} \quad (19)$$

where

$$\mathcal{N} = \bar{u}(k') \gamma_\alpha (1 - \gamma_5) u(k) (V^\alpha - A^\alpha), \quad (20)$$

$\Lambda(p) = \not{p} + M$ . In eq. (19) the projection operator  $(g^{\rho\sigma} - p'^\rho p'^\sigma / M^2)$  guarantees the condition  $(\xi \cdot p') = 0$ ,

The vector  $\xi^\alpha$  can be decomposed along the following three independent 4-vectors  $Q_i^\alpha$  orthogonal to  $p'^\alpha$ :

$$Q_+^\alpha = k_+^\alpha - \frac{(p' k_+)}{M^2} p'^\alpha, \quad Q_-^\alpha = k_-^\alpha - \frac{(p' k_-)}{M^2} p'^\alpha, \quad Q_p^\alpha = p^\alpha - \frac{(p' p)}{M^2} p'^\alpha \quad (21)$$

where

$$k_+ = (k + k'), \quad k_- = (k - k') = q \quad (22)$$

After standard calculations we obtain:

$$\xi^\alpha = \frac{m}{(kp) J_0} [Q_+^\alpha P_+ + Q_-^\alpha P_- + Q_p^\alpha P_p] \quad (23)$$

$$P_+ = -[y G_M^{CC} + (2 - y) G_A] G_E^{CC} \quad (24)$$

$$P_- = G_A [y G_M^{CC} + (2 - y) G_A] - F_2^{CC} [(2 - y) \tau G_M^{CC} + y(1 + \tau) G_A] \quad (25)$$

$$P_p = -\frac{F_2^{CC}}{y} [2y(2 - y) \tau G_M^{CC} + [2\tau[1 + (1 - y)^2] + y^2] G_A] \quad (26)$$

Here

$$\begin{aligned} G_E^{CC} &= F_1^{CC} - \tau F_2^{CC}, & G_M^{CC} &= F_1^{CC} + F_2^{CC} \\ J_0 &= \frac{Tr [\mathcal{N}\Lambda(p)\bar{\mathcal{N}}\Lambda(p')]}{8^2(kp)^2}, & y &= \frac{(pq)}{(pk)}, & \tau &= \frac{Q^2}{4M^2} \end{aligned} \quad (27)$$

From (15) it follows:

$$G_M^{CC} = G_M^p - G_M^n, \quad G_E^{CC} = G_E^p - G_E^n, \quad (28)$$

where  $G_M^{p,n}$  and  $G_E^{p,n}$  are the magnetic and charge form factors of proton and neutron.

From (23) we easily find the polarization vector of the proton in the laboratory frame. We have:

$$\vec{\xi} = \frac{1}{J_0 E} \left\{ (\vec{k} + \vec{k}') P_+ + \vec{q} \left[ -\frac{E + E'}{M} P_+ + \left(1 + \frac{E - E'}{M}\right) (P_- - P_p) \right] \right\}, \quad (29)$$

Here  $E$  and  $E'$  are energies of neutrino and final muon,

$$E' = \frac{E}{1 + (2E/M) \sin^2 \frac{\theta}{2}}, \quad y = \frac{E - E'}{E}, \quad (30)$$

$\theta$  is the angle between vectors  $\vec{k}$  and  $\vec{k}'$ .

The polarization vector lays in the scattering plane.<sup>1</sup> For the longitudinal  $\xi_{\parallel}$  and transverse  $\xi_{\perp}$  components of the polarization we have:

$$\vec{\xi} = \xi_{\perp} \vec{e}_{\perp} + \xi_{\parallel} \vec{e}_{\parallel}, \quad (31)$$

where  $\vec{e}_{\perp}$  and  $\vec{e}_{\parallel}$  are two orthogonal unit vectors in the scattering plane:

$$\vec{e}_{\parallel} = \frac{\vec{p}'}{|\vec{p}'|} = \frac{\vec{q}}{|\vec{q}|}, \quad \vec{e}_{\perp} = \vec{e}_{\parallel} \times \vec{n}, \quad \vec{n} = \frac{\vec{k} \times \vec{k}'}{|\vec{k} \times \vec{k}'|}. \quad (32)$$

From (29), (31) and (32) we obtain:

$$s_{\perp} = \xi_{\perp} = \left( \frac{1}{J_0} \right) \frac{-2 E' \sin \theta}{|\vec{q}|} [G_A(2 - y) + G_M^{CC} y] G_E^{CC}. \quad (33)$$

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<sup>1</sup>It is obvious that the component orthogonal to the scattering plane disappears due to  $T$ -invariance.



$$\begin{aligned}
s_{\parallel} &= \frac{M}{p'_0} \xi_{\parallel} = \\
&= \frac{1}{J_0} \frac{q_0}{|\vec{q}|} [G_A(2-y) + G_M^{CC} y] \left[ G_M^{CC}(2-y) + G_A(y + \frac{2M}{E}) \right]. \quad (34)
\end{aligned}$$

Here  $s_{\parallel}$  and  $s_{\perp}$  are the longitudinal and transverse components of the polarization vector in the rest frame of the recoil nucleon:

$$s^{\alpha} = (0; s_{\parallel}, s_{\perp}) \quad (35)$$

$M/p'_0$  is the Lorentz boost along  $\vec{p}'$ .

Let us note that at  $G_A = 0$ , using the kinematic relations

$$|\vec{q}| = Ey \sqrt{\frac{1+\tau}{\tau}}, \quad \frac{q_0}{|\vec{q}|} = \sqrt{\frac{\tau}{1+\tau}}, \quad (36)$$

one can show that eqs. (33) and (34) coincide with the well known expressions for the transverse and longitudinal polarizations of the recoil protons in elastic scattering of longitudinally polarized leptons on unpolarized protons (see, for example, [1]).

Taking into account that the hadronic part of the processes  $\nu_{\mu} + n \rightarrow \mu^{-} + p$  and  $\bar{\nu}_{\mu} + p \rightarrow \mu^{+} + n$  are the same, we easily obtain the polarizations of the final nucleons for both processes:

$$(J_0 s_{\perp})^{\nu, \bar{\nu}} = \frac{-2E' \sin \theta}{|\vec{q}|} [\pm y G_M^{CC} + (2-y)G_A] G_E^{CC} \quad (37)$$

and

$$(J_0 s_{\parallel})^{\nu, \bar{\nu}} = \frac{q_0}{|\vec{q}|} [\pm y G_M^{CC} + (2-y)G_A] \left[ (2-y) G_M^{CC} \pm \left( y + \frac{2m}{E} \right) G_A \right]. \quad (38)$$

Here  $J_0^{\nu, \bar{\nu}}$  is determined from the differential cross section:

$$J_0^{\nu, \bar{\nu}} = \frac{d\sigma^{\nu, \bar{\nu}}}{dQ^2} \cdot \frac{2\pi}{G_F^2 \cos^2 \theta_c} \quad (39)$$

In terms of the form factors it is given by the expression:

$$\begin{aligned}
J_0^{\nu, \bar{\nu}} &= 2(1-y) \left[ G_A^2 + \frac{\tau(G_M^{CC})^2 + (G_E^{CC})^2}{1+\tau} \right] + \frac{my}{E} \left[ G_A^2 - \frac{\tau(G_M^{CC})^2 + (G_E^{CC})^2}{1+\tau} \right] \\
&\quad + y^2 (G_M^{CC} \pm G_A)^2 \mp 4y G_M^{CC} G_A \quad (40)
\end{aligned}$$

## 4 Comments

– From eq.(37) we obtain a rather simple expression for  $G_A$ :

$$G_A = \frac{-1}{2-y} \left\{ \frac{M \sqrt{\tau(1+\tau)}}{E' \sin \theta} \frac{(J_0 \cdot s_\perp)^{\nu, \bar{\nu}}}{G_E^{CC}} \pm y G_M^{CC} \right\} \quad (41)$$

– Note, that the electric form factor does not enter eq.(38). Thus the axial form factor  $G_A$  is determined only by the cross section, the longitudinal polarization  $\xi_\parallel$  and the best known magnetic form factors of the proton and neutron.

– If the neutrino detector is in a magnetic field, then both the transverse and longitudinal polarizations could be measured (like in the case of elastic  $e-p$  scattering). For their ratio we have:

$$\left( \frac{s_\parallel}{s_\perp} \right)^{\nu, \bar{\nu}} = \frac{-q_0}{2E' \sin \theta} \frac{[(2-y) G_M^{CC} \pm G_A(y + 2M/E)]}{G_E^{CC}} \quad (42)$$

Then for the axial form factor we obtain:

$$G_A = \mp \frac{E + E'}{E + E' + 2m} \left[ \frac{4EE' \sin \theta}{E^2 - E'^2} G_E^{CC} \left( \frac{s_\parallel}{s_\perp} \right)^{\nu, \bar{\nu}} + G_M^{CC} \right] \quad (43)$$

– Finally, let us notice the relations:

$$(J_0 s_\perp)^\nu + (J_0 s_\perp)^{\bar{\nu}} = \frac{-4 E' \sin \theta}{|\vec{q}|} (2-y) G_A G_E^{CC} \quad (44)$$

$$(J_0 s_\parallel)^\nu + (J_0 s_\parallel)^{\bar{\nu}} = \frac{q_0}{|\vec{q}|} 4 G_A G_M^{CC} \left\{ [1 + (1-y)^2] + \frac{My}{E} \right\} \quad (45)$$

## 5 Conclusion

Investigation of the CCQE neutrino processes and determination of the axial form factor of the nucleon is of great importance for the theory and for the modern high-precision neutrino oscillation experiments. Many experiments on measurement of the cross sections of the CCQE neutrino processes in a wide range of neutrino energies have been done. From analysis of the data of these experiments the value of the parameter  $M_A$ , which determines

the  $Q^2$ -behavior of the axial form factor in the dipole approximation, was determined. Usually in such analysis the impulse approximation for the target nuclei is used. The values of  $M_A$  determined from the data of the different experiments in such a way are not compatible. There could be different reasons for such a disagreement: nuclei effects, more complicated than dipole  $Q^2$ -dependence of the axial form factor etc.

In this paper we present the calculation of the polarization of the final nucleon in CCQE scattering. Relations that express the axial form factor through the polarization of the final nucleon and the electromagnetic form factors are obtained. Experiments on measurement of the polarization of the final proton in elastic scattering of longitudinally polarized electrons on unpolarized protons drastically changed our understanding about the electromagnetic form factors of the proton. Analogously, we suggest that measurement of the polarization of the final nucleon in CCQE processes will provide additional information about the axial form factor. It is obvious that such measurement is a challenge. However, taking into account the importance of the problem of the axial form factor and the rapid progress of the neutrino detection technique it is worth to consider such a possibility.

## Acknowledgments

The work of E.Ch. was partially supported by a priority Grant between JINR-Dubna and the Republic of Bulgaria on theme 01-3-1070-2009/2013 of the BLTP.

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